Game Theory

BA252
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Assumptions:

- opponents are rational, intelligent, and act in their own self interest.
- Rules and outcomes are known. Each player decides on his own strategy with regard to the opponents' projected response and the resulting expected outcomes.

Definitions:

- ***Games*** -differ from other decision theory in that the events which are not under our control are under the control of a sensate opponent who is assumed to be rational - acting in his own self-interest. This contrasts with "nature" which was assumed to act with some definable probabilities but without prejudice.
- ***Zero Sum game*** -game in which the sum of all players' gains and losses is zero. That is, any player's gain must be balanced by some other player's loss. The size of the pie to be divided doesn't change.
- ***Nonzero sum game*** -the wealth to be divided among the players changes in response to the strategies of the players. In a two player game, both players can be winners, or both players can be losers. The essence of the prisoner's dilemma is that the stable strategy is one in which both players are losers.
- ***Cooperative model*** -Players agree to split the winnings at the end of the game, so they act in agreement to achieve the outcome that maximizes total wealth. (max profit, min loss).
- ***Win-win negotiation*** -outcome in which the settlement results in an increase in the total wealth, with each party improving his condition through the exchange.
- ***Free trade -***Exchange in which both parties gain (increase wealth). Since players have the option of not playing, or playing another game, the exchange won't take place unless both players win.
- ***Competitive model*** -Each player independently acts to maximize his own winnings without regard to the total wealth at the end.
- ***Compromise*** -process by which someone who isn't going to get his way makes sure no one else does either.

A Zero Sum Game with a stable "saddle" solution:

by convention, the table shows payoffs for player A, whose alternative strategies are listed down the left hand side. Alternative strategies for player B are listed across the top of the table. Gains for A are losses for B.

Values in the final row are the highest gains A could choose if B pursued each of the strategies, X, Y, or Z. Thus, if B elected strategy X, A would choose strategy 1. If B pursued Y, A would choose 1, and if B chose Z, A would still choose 1. Each choice of A is enclosed in a square.

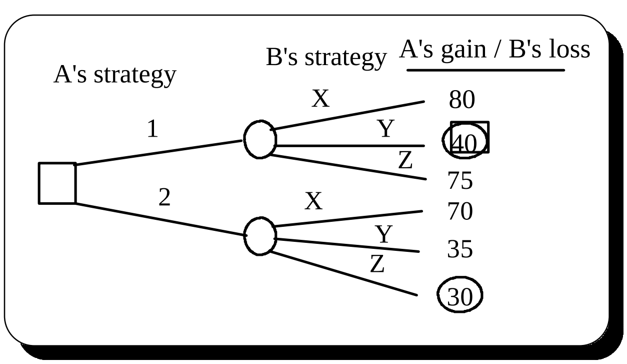
Values in the final column are the minimum losses B could choose for each strategy of A (each is circled). If A pursued 2, then B would choose the lowest loss strategy, Z. If A chooses 1, B would choose Y.

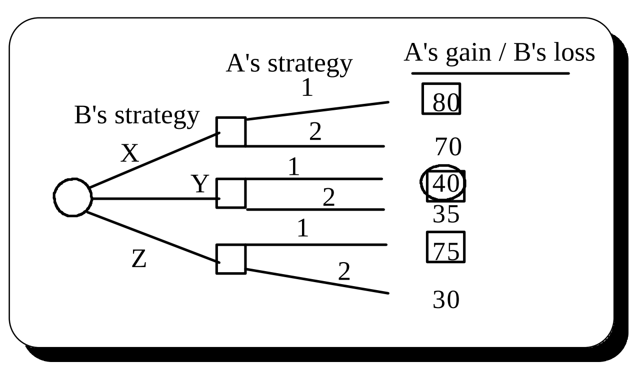
The stable solution is for A to choose 1, and B to choose Y. This is the minimum of A's maximum gains for each of B's strategies(Maximin for A), and the Maximum of B's minimum losses for each of A's strategies (Minimax for B). This is called a "saddle point solution" because -viewed as profits -- it's on minimum point for both the A and B Axes -like a saddle.

A \ B	Strategy	Strategy Y	Strategy	Min
strategy 1	80	40	75	40 B's
Strategy 2	70	35	30	30
Max Gain	80	40	75	
	A's			

Another way to look at this is as a tree diagram:

A, in planning his strategy, would consider what B would do in response and what the resulting payoff would be. A then chooses the strategy that would result in the best final outcome. B does the same thing. Thus, in this case, A would pick the strategy that results in the maximum of the minimum gains. The solution is stable because, with these particular numbers, neither A nor B has incentives to switch to another strategy.





Zero sum games requiring mixed strategies:

A's market share as a function of advertising strategy

A / B	None	Medium	Large	B's min loss
none	60	50	(40)	40
medium	70	[70]	(50)	50
large	[80]	(60)	[75]	60
A's max gain	80	70	75	

Not all games have equilibrium points with simple strategies. In the following example, if B knew that A was going to choose no advertising, then B would max it's market share by pursuing large advertising. However, if A could predict that B would do large advertising, Then A would also do large advertising. But, if B could predict that A would do large advertising, then B would choose to do medium advertising, and so forth. Since there isn't any combination of strategies that both would pursue simultaneously, there isn't any stable simple strategy. For any strategy A would pursue, B would prefer a strategy that would cause A to prefer a different strategy. One solution is for players to pursue random strategies to keep opponents from reacting effectively. (Assume opponents have to choose strategies ahead of time) . However, even in random mixed strategies, if the probabilities are known, the competitor can still gain an advantage by treating it as a max EMV decision problem. To avoid this, Each opponent should choose mixed strategies with probabilities that would make the opponent indifferent - no advantage can be gained by pursuing one strategy versus another. Predictable pure strategies would result in lower gains than using mixed strategies with probabilities chosen to make the opponent indifferent. First, to simplify the matrix, let's eliminate strategies that A & B wouldn't pursue in any case. No matter what B would do, A wouldn't pursue the strategy of no advertising. Likewise, for all strategies of A, B would do some advertising. Eliminating the strategies of no advertising on the parts of both A and B reduces the problem to the 2 by 2 matrix shown. Now, suppose A chose to do medium advertising. B, knowing this, would choose to do a large amount of advertising to minimize A's market share. A's payoff would be 50% market share. Likewise, if B knew that A was going to pursue a strategy of large advertising, B would follow the strategy of medium advertising, and A's share would be 60%. From the other side, suppose B were to pursue a pure strategy of medium advertising. A, knowing this, would choose a strategy of medium advertising as well, giving B a market share of only 30% (100%-70%). If B consistently did large advertising, A would also do large advertising, and B's market share would be only 25%. The only way to get equilibrium is for each to pursue strategies in unpredictable patterns with probabilities such that the other guy is indifferent between his alternatives (expected values are the same for each alternative).

Solving for the probabilities required:

A's EMV's:

EMV medium for A = EMV large for A

$70q + 50(1-q) = 60q + 75(1-q)$

Solving for q gives $q = 5/7$;

so $(1-q) = 2/7$.

Therefore: if B pursues medium 5/7 of the time and large 2/7 of the time, A will be indifferent between his own strategies of medium or large advertising and will have an expected return of 64 and 2/7% market share. This is better than the 60% obtainable from a pure strategy of large advertising.

Matrix Reduced by eliminating dominated strategies

A / B	medium	large	proportion
medium	70	50	p
large	60	75	1 - p
proportion	q	1- q	

B's EMV's:

EMV medium for B = EMV large

$70p + 60(1-p) = 50p + 75(1-p)$

Solving for p gives $p = 3/7$, so $(1-p) = 4/7$

therefore, if A pursues the medium strategy 3/7 of the time, B will have the same expected payoff (B's market share = 100% - 64 and 2/7% = 35 and 5/7%) for both strategies and will be indifferent between them. This payoff is better than the best pure strategy result of 30%. The combined result is stable. Neither party has incentive to change.

A Nonzero-Sum Game and The Prisoner's Dilemma: Now, picture two partners in crime who have been picked up by the police. The penalty for robbery is 10 years in jail. If neither confesses, then the worst they can be convicted of is stealing the car they used for the getaway, which has a penalty of one year in prison. If, on the other hand, they both confess, they will be convicted of the robbery as well, and will get 5 years in prison as a reduced penalty for being cooperative with the police. The District attorney, in order to split the team and provide incentive for confession, has offered each separately to let him off with a suspended sentence if he confesses and his partner doesn't. In the event one of the partners is convicted without being cooperative (confessing) he will get the full penalty of 10 years in prison. The matrix shows the different payoffs in this game in which the total penalties change depending on the combination of strategies. A's outcome is in the upper left corner of each split cell, and B's outcome is shown in the lower right corners. To minimize the total penalty, both should agree to stay quiet and not confess. In this case, the total penalty would be 2 years in prison, one for each. However, if each considers his best strategy in case of each strategy that might be pursued by his opponent (former partner), it's apparent that he would achieve the best result in each case by confessing. A's best strategy if B is quiet is to confess and get off. A's best strategy if B confesses is to confess as well so that his sentence will be reduced. Thus, regardless of B's strategy, A's best strategy is to confess. The same holds true for B's options. The strategy of staying quiet is dominated by the strategy of confessing - which leads to the inevitable result that both will be convicted and sent up the river for 5 years each. The prisoner's dilemma is whether or not to trust his colleague with the hope of a 1 year penalty or to pursue his dominant strategy of confessing --betraying his partner -- in the hopes that his colleague trusted him not to do so. Research on simulated conditions of this sort has shown that game players who take a chance on the good faith of their fellows and follow the no-confession strategy are consistently exploited by their less-trusting partners. Consider how this process applies to everyday games such as nuclear armaments, advertising costs, cartels and price fixing arrangements. It's a tough life. Never trust anybody.

A / B	hold quiet	Confess	B's Min penalty
Hold Quiet	-1 / / -1	-10 / / (0)	0
Confess	[0] / / -10	[-5] / / (-5)	-5
A's Min	0	-5	

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